

TEST PREP WED. - 2 UNDETERMINED COEF. PROBLEMS

3.5 = UNDETERMINED COEF. (CONTINUED)

ENTRY TASK GIVE THE FORM OF A PARTICULAR SOLN TO

1) y'' - y = cos(2t) INITIAL GUESS -> A cos(2t) + B sin(2t)

2) y'' - y = e^{3t} -> A e^{3t}

3) y'' - y = t e^{3t} -> (At + B) e^{3t}

4) y'' - y = e^t -> A e^t -> A t e^t

5) y'' + y = sin(t) -> A cos(t) + B sin(t) -> A t cos(t) + B t sin(t)

1-4) HOMOGENEOUS SOLN r^2 - 1 = 0 => r = +/- 1 y1 = e^t, y2 = e^-t

ARE ANY OF THOSE INITIAL GUESSES OF THIS FORM IF NO, THEN FORM WILL WORK. IF YES, MULTIPLY BY t.

5) HOMOGENEOUS SOLN r^2 + 1 = 0 => r = +/- i y1 = cos(t), y2 = sin(t)

MULT. BY t

SEE HANDOUT FOR MORE PRACTICE!!!

Ex) $y'' + 3y' - 4y = 4t - 5e^{-4t}$, $y(0) = 1, y'(0) = 0$

I Homogeneous Sol'n:

$r^2 + 3r - 4 = 0 \Rightarrow (r+4)(r-1) = 0$

$r_1 = -4, r_2 = 1$

$y_1 = e^{-4t}, y_2 = e^t$

II Particular Sol'n

$4t - 5e^{-4t}$
 $A t + B \quad C e^{-4t}$

Homogeneous sol'n!
MULT. BY t

$A t + B \quad C t e^{-4t}$

$y_p = A t + B + C t e^{-4t}$

$y_p' = A + C e^{-4t} - 4 C t e^{-4t}$

$y_p'' = 0 - 4 C e^{-4t} - 4 C e^{-4t} + 16 C t e^{-4t}$

substitute:

$(-8 C e^{-4t} + 16 C t e^{-4t}) + 3(A + C e^{-4t} - 4 C t e^{-4t}) - 4(A t + B + C t e^{-4t})$

$= -4 A t + 3 A - 4 B + e^{-4t} [-8 C + 16 C t + 3 C - 12 C t - 4 C t]$

$= (-4 A) t + (3 A - 4 B) + (-5 C) e^{-4t}$

$\begin{matrix} \text{---} \\ \text{---} \\ \text{---} \end{matrix} \begin{matrix} 4 t & & - 5 e^{-4t} \end{matrix}$

$-4 A = 4 \Rightarrow A = -1$

$3 A - 4 B = 0 \Rightarrow 4 B = 3 A \Rightarrow B = -3/4$

$-5 C = -5 \Rightarrow C = 1$

$$Y_p = -t - \frac{3}{4} + te^{-4t}$$

General Sol'n

$$y = c_1 e^{-4t} + c_2 e^t + Y_p(t)$$

ASIDE

CHECK $\therefore y = -t - \frac{3}{4}$

$$y' = -1$$

$$y'' = 0$$

$$y'' + 3y' - 4y :$$

$$0 + 3(-1) - 4(-t - \frac{3}{4}) = 4t \checkmark$$

$$\bullet y = te^{-4t}$$

$$y' = e^{-4t} - 4te^{-4t}$$

$$y'' = -4e^{-4t} - 4e^{-4t} + 16te^{-4t} = -8e^{-4t} + 16te^{-4t}$$

$$y'' + 3y' - 4y = (-8e^{-4t} + 16te^{-4t}) + 3(e^{-4t} - 4te^{-4t}) - 4te^{-4t} = e^{-4t}(-8 + 16t + 3 - 12t + 4t) = -5e^{-4t} \checkmark$$

INITIAL CONDITIONS

$$y(0) = 1 \Rightarrow c_1 + c_2 - (0) - \frac{3}{4} + (0) = 1$$

$$\Rightarrow c_1 + c_2 = \frac{7}{4}$$

$$y'(t) = -4c_1 e^{-4t} + c_2 e^t - 1 + e^{-4t} - 4te^{-4t}$$

$$y'(0) = 0 \Rightarrow -4c_1 + c_2 - 1 + 1 = 0$$

$$\Rightarrow -4c_1 + c_2 = 0 \quad c_2 = 4c_1$$

$$c_1 + 4c_1 = \frac{7}{4} \Rightarrow 5c_1 = \frac{7}{4} \Rightarrow c_1 = \frac{7}{20}$$

$$c_2 = 4c_1 = 4 \cdot \frac{7}{20} = \frac{7}{5}$$

$$y(t) = \frac{7}{20} e^{-4t} + \frac{7}{5} e^t - t - \frac{3}{4} + te^{-4t}$$